

Progression in Calculations



Understanding and Using Calculations

For all calculations, children need to:

- Understand the = sign as **is the same as**, as well as **makes**.
- See calculations where the equals sign is in different positions and what these number sentences represent, e.g. $3 + 2 = 5$ and $5 = 7 - 2$.
- Decide on the most appropriate method i.e. mental, mental with jottings, written method or calculator
- Approximate before calculating and check whether their answer is reasonable.

Addition

Children need to understand the concept of addition, that it is:

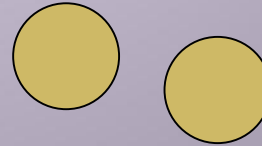
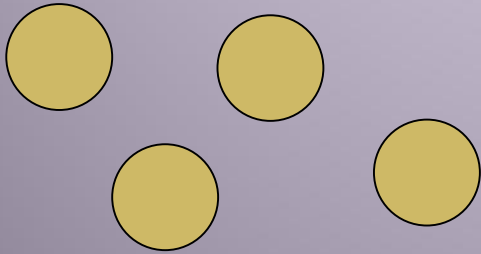
- **Combining two or more groups to give a total or sum**
- **Increasing an amount**

They also need to understand and work with certain principles:

- **Inverse of subtraction**
- **Commutative i.e. $5 + 3 = 3 + 5$**
- **Associative i.e. $5 + 3 + 7 = 5 + (3 + 7)$**

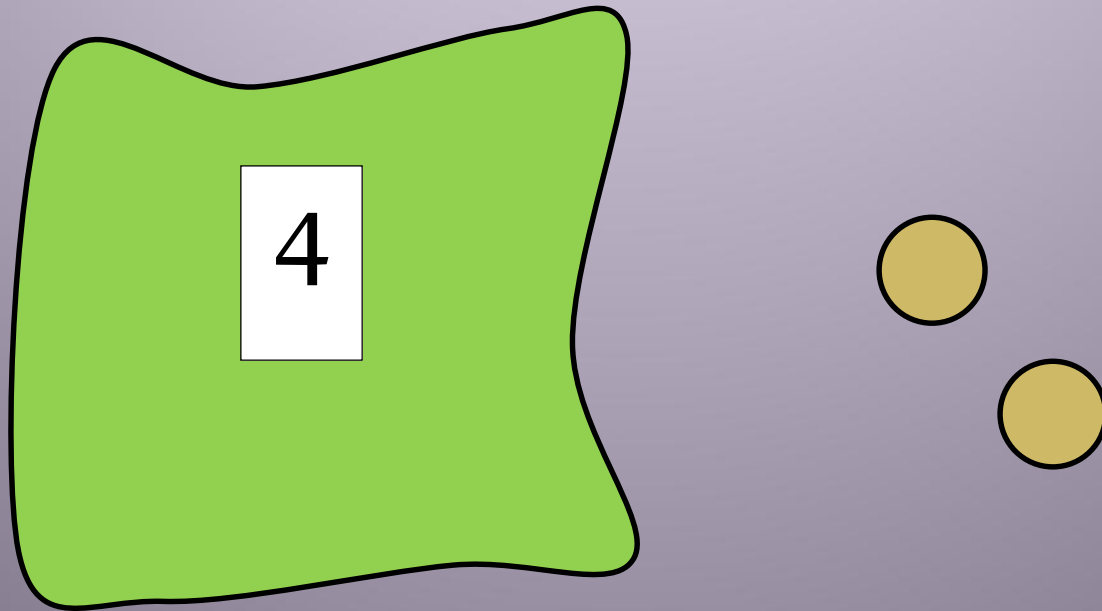
Counting All

Using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total, e.g. $4 + 2$



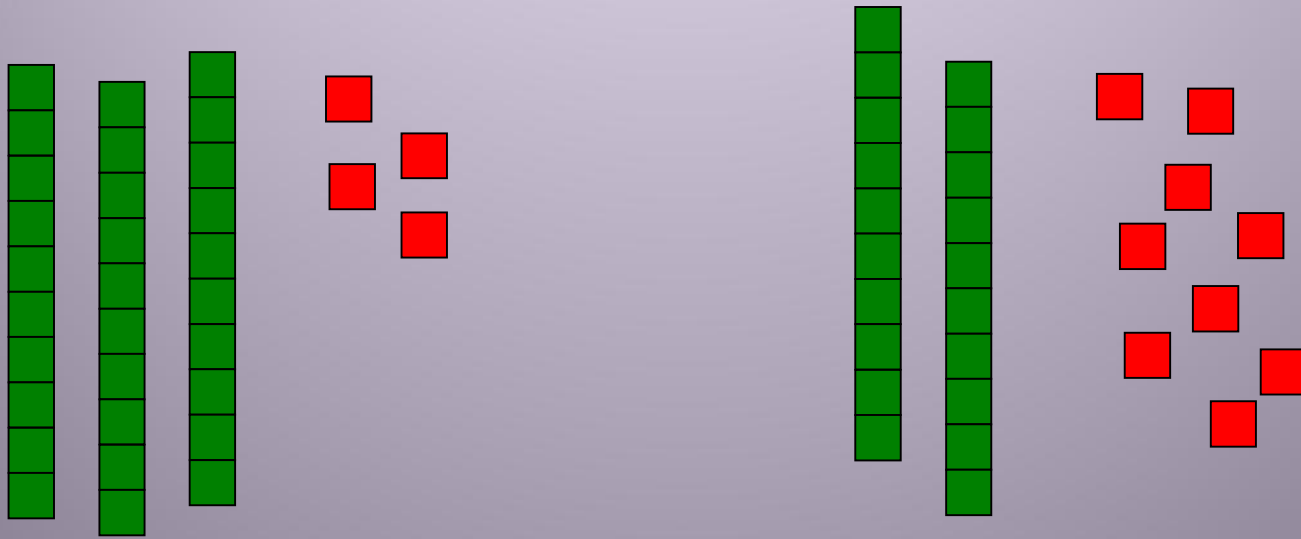
From Counting All to Counting On

To support children in moving from counting all to counting on, have two groups of objects but cover one so that it can not be counted, e.g. $4 + 2$



Adding Two Digit Numbers

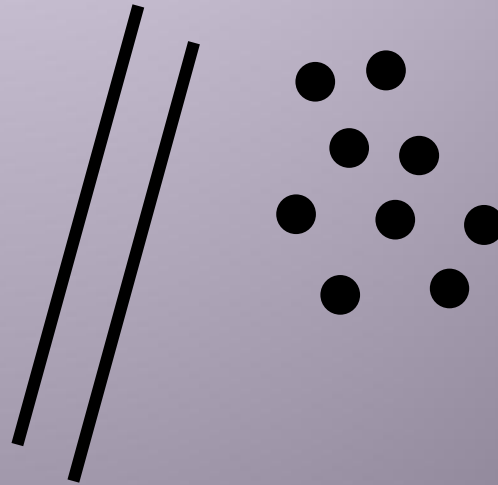
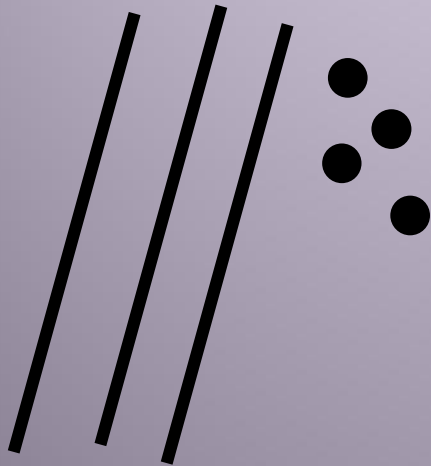
Children can use base 10 equipment to support their addition strategies by basing them on counting, e.g. $34 + 29$



Children need to be able to count on in 1s and 10s from any number and be confident when crossing tens boundaries.

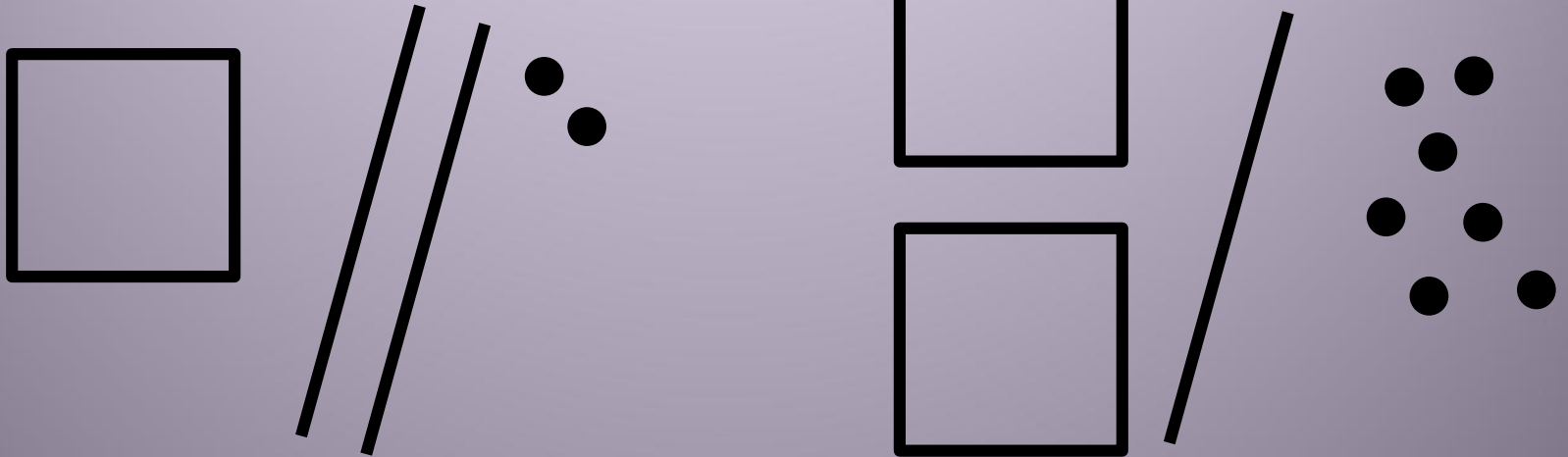
Adding Two Digit Numbers

Children can support their own calculations by using jottings, e.g.
 $34 + 29$



Adding Three Digit Numbers

Children can support their own calculations by using jottings, e.g.
 $122 + 217$



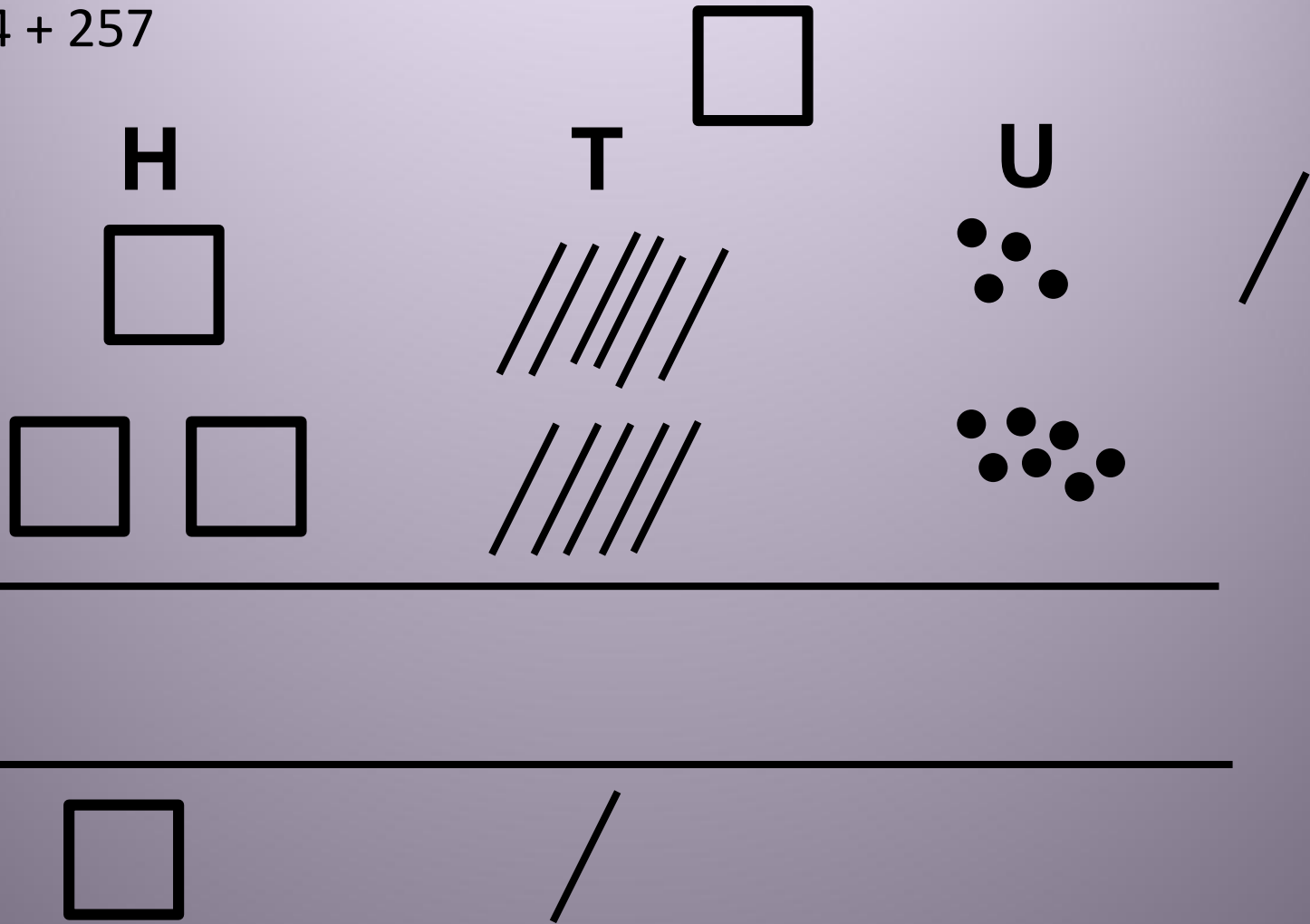
Beginning Column Addition

$$\begin{array}{r} \text{T U} \\ 67 \\ + 24 \\ \hline 11 \quad \bullet\bullet\bullet\bullet\bullet \quad \bullet\bullet \\ 80 \quad // // // // // \quad // \\ \hline 91 \\ \hline \end{array}$$

The diagram illustrates the process of adding 67 and 24. The numbers are aligned by place value: Tens (T) and Units (U). The addition is shown in three rows. The first row shows the numbers 67 and 24 with a plus sign. A horizontal line is drawn below the numbers. The second row shows the result of adding the units: 7 + 4 = 11. To the right of the '11' are ten dots arranged in two groups of five, representing the ten units from the sum. The third row shows the result of adding the tens: 6 + 2 = 8. To the right of the '80' are eight slanted lines, representing the eight tens from the sum. A horizontal line is drawn below the '80'. The final row shows the final result: 91. A horizontal line is drawn below the '91'.

Continuing Column Addition

e.g. $164 + 257$



Efficient Column Addition

$$\begin{array}{r} \text{HTU} \\ 164 \\ + 257 \\ \hline 421 \\ \hline 11 \end{array}$$

Subtraction

Children need to understand the concept of subtraction, that it is:

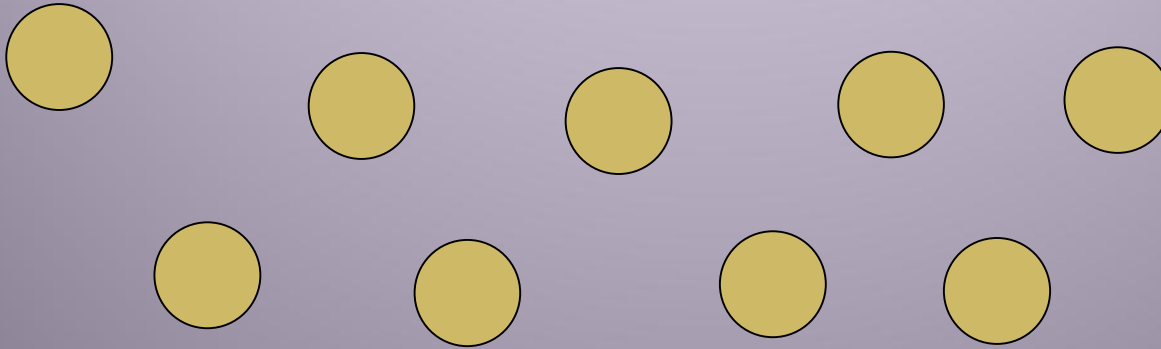
- **Removal of an amount from a larger group (take away)**
- **Comparison of two amounts (difference)**

They also need to understand and work with certain principles:

- **Inverse of addition**
- **Not commutative i.e. $5 - 3 \neq 3 - 5$**
- **Not associative i.e. $(9 - 3) - 2 \neq 9 - (3 - 2)$**

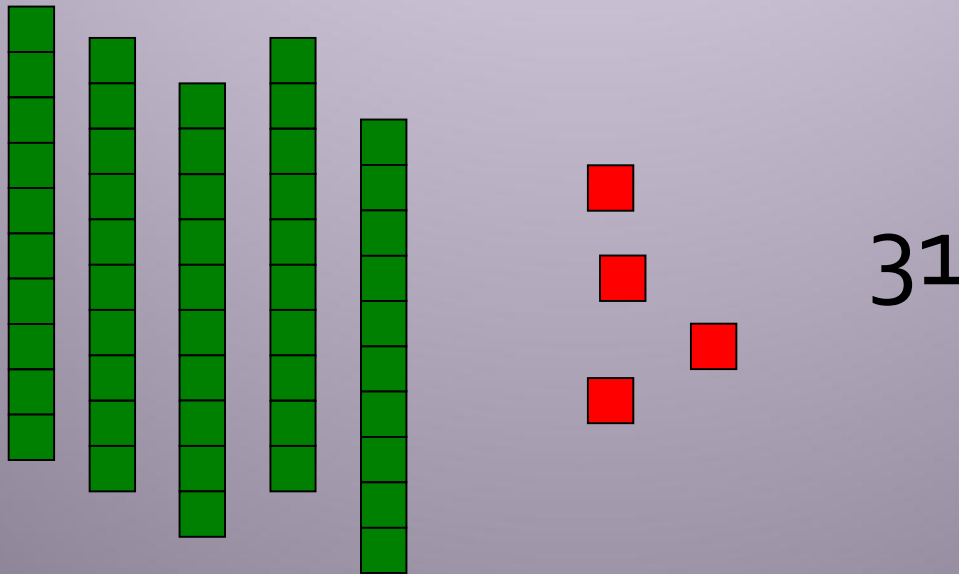
Taking Away

Using practical equipment to count out the first number and removing or taking away the second number to find the solution, e.g. $9 - 4$



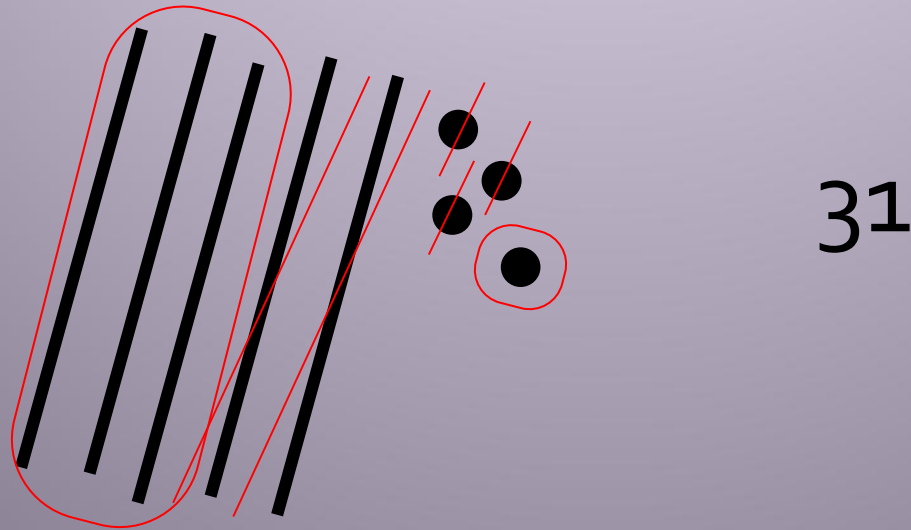
Taking Away Two Digit Numbers

Children can use base 10 equipment to support their subtraction strategies by basing them on counting, e.g. $54 - 23$



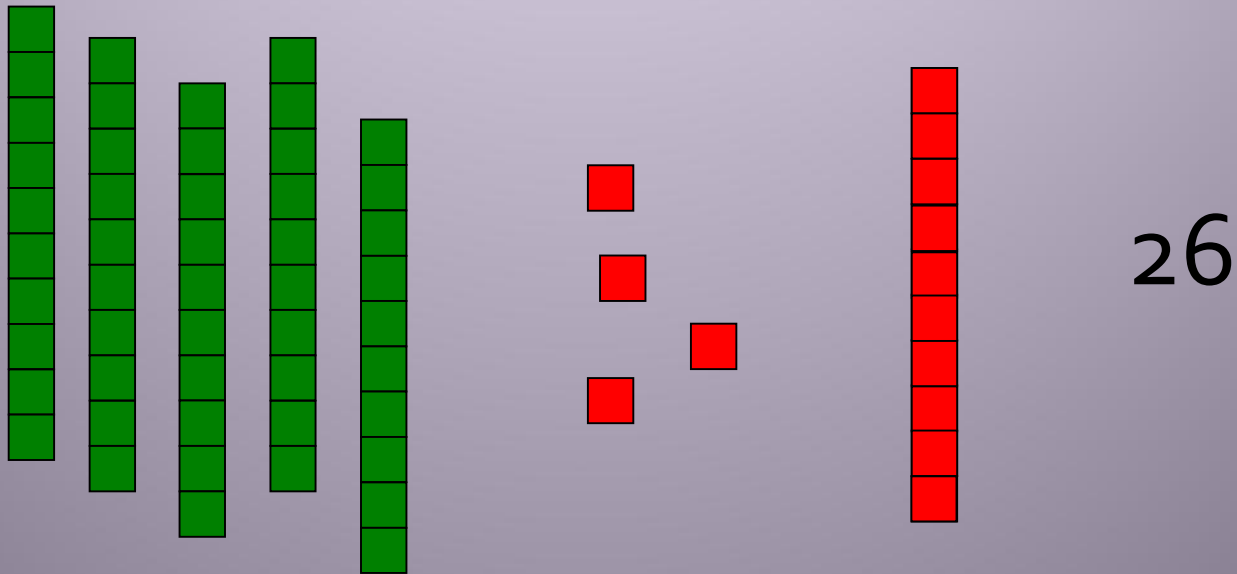
Taking Away Two Digit Numbers

Children can support their own calculations by using jottings, e.g.
54 - 23



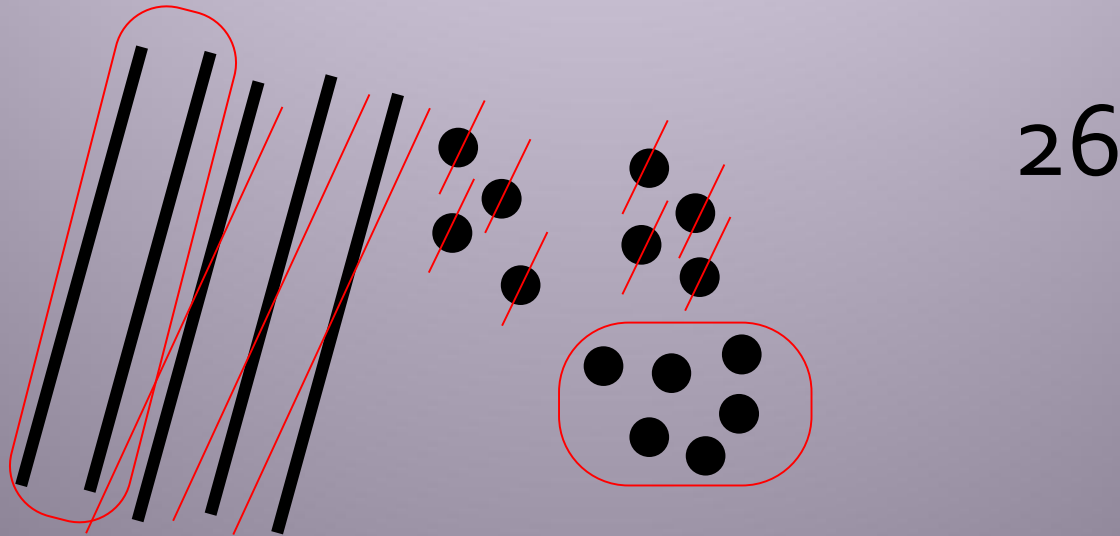
Taking Away Two Digit Numbers (Exchange)

Children can use base 10 equipment to support their subtraction strategies by basing them on counting, e.g. $54 - 28$

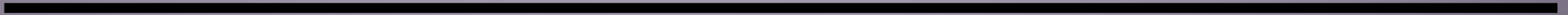
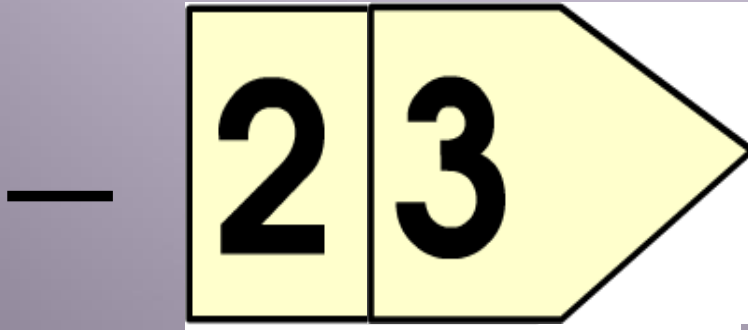
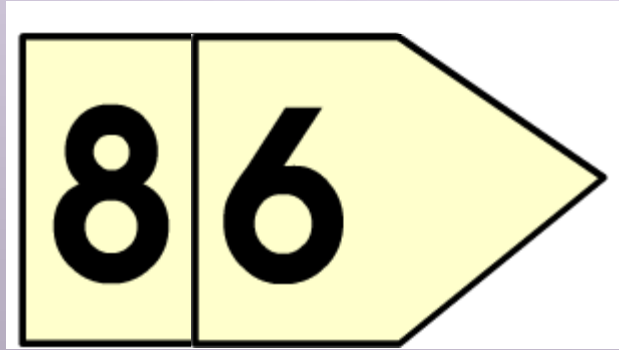


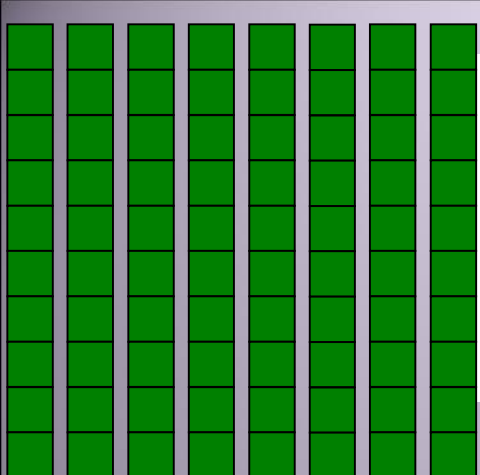
Taking Away Two Digit Numbers (Exchange)

Children can support their own calculations by using jottings, e.g.
54 - 28

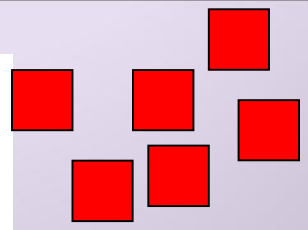


Beginning Column Subtraction





80

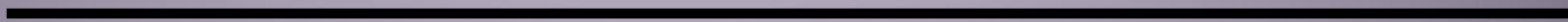


6

-

20

3

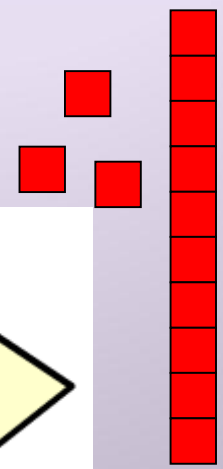
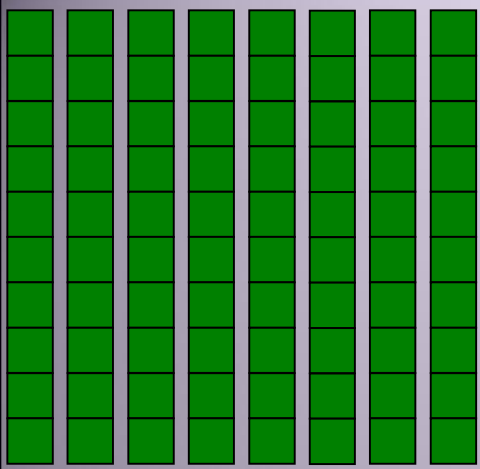


60

3

Beginning Column Subtraction (Exchange)

83
- 26



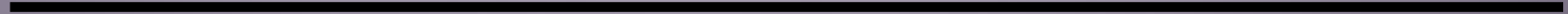
70

13

-

20

6

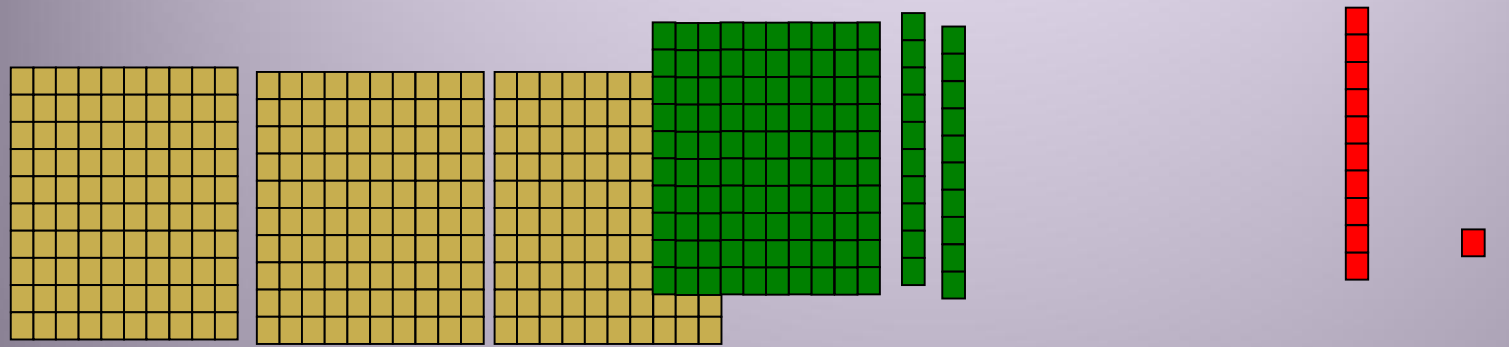


50

7

Continuing Column Subtraction

e.g. 321 - 157



H	T	U	
200	110	11	
300	20	7	
- 100	50		
100	60	4	= 164

Efficient Decomposition

HTU

²
~~3~~ ¹¹
~~2~~ ¹
~~1~~

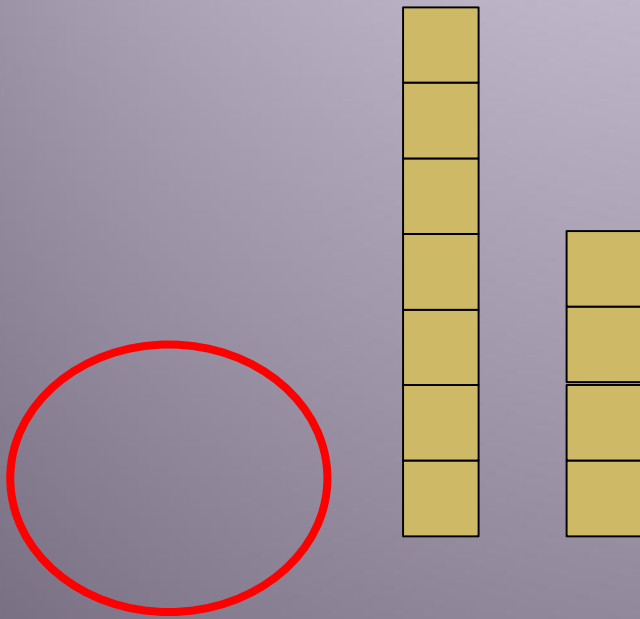
- 1 5 7

1 6 4

Finding the Difference (Counting Back)

Children need to understand how counting back links to subtraction,
e.g. $7 - 4$

Make the large tower the same size as the small tower.



Finding the Difference (Counting On)

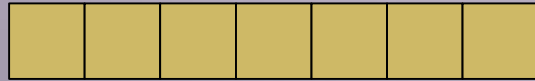
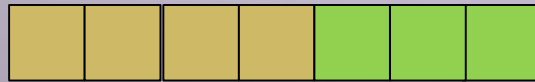
Children need to understand how counting on links to subtraction, e.g.
 $7 - 4$

Make the small tower the same size as the large tower.



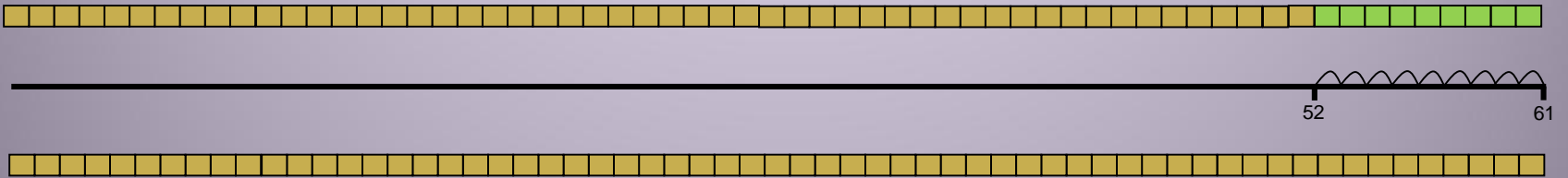
Finding the Difference (Counting On)

To begin linking to number lines, this can be looked at horizontally instead of vertically.

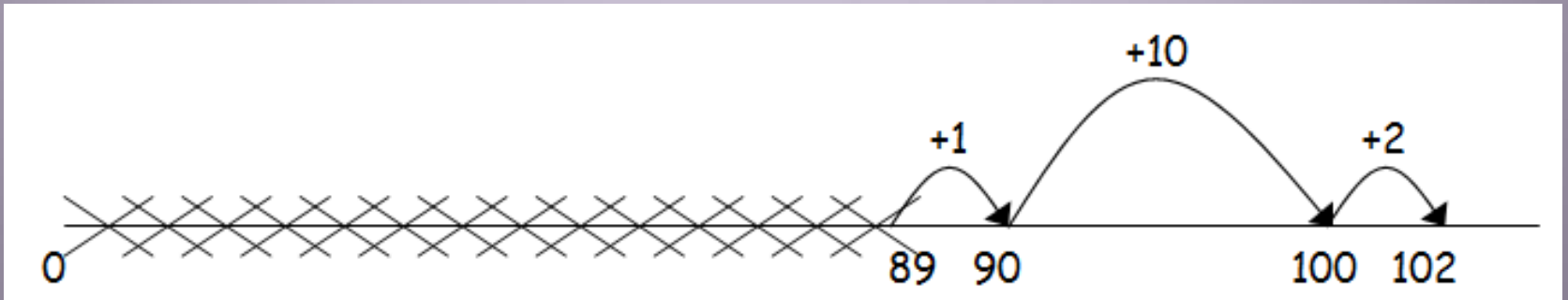


Moving on to Number lines

61 - 52



Consolidating Number Lines



Multiplication

Children need to understand the concept of multiplication, that it is:

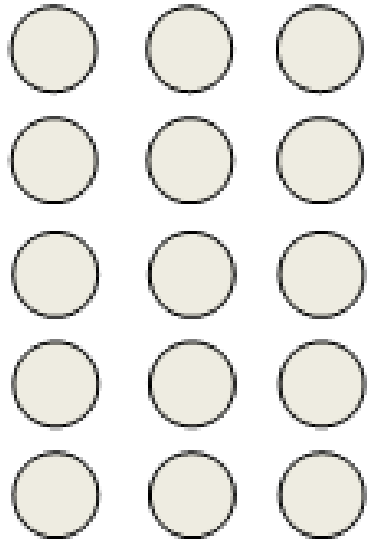
- **Repeated addition**
- **Can be represented as an array**

They also need to understand and work with certain principles:

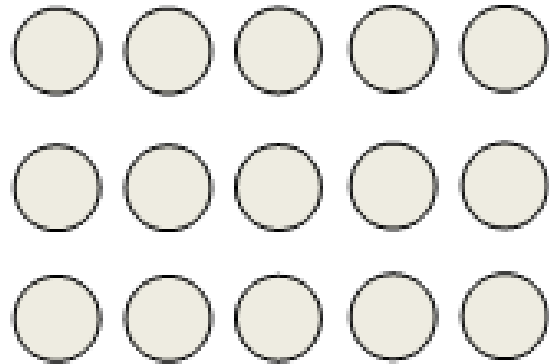
- **Inverse of division**
- **Is commutative i.e. $3 \times 5 = 5 \times 3$**
- **Is associative i.e. $2 \times (3 \times 5) = (2 \times 3) \times 5$**

Use of Arrays

Children need to understand how arrays link to multiplication through repeated addition and be able to create their own arrays.



$$3 + 3 + 3 + 3 + 3 = 15$$



$$5 + 5 + 5 = 15$$

Continuation of Arrays

Creating arrays on squared paper (this also links to understanding area).

$$4 \times 7 =$$

x	x	x	x	x	x	x	
x	x	x	x	x	x	x	
x	x	x	x	x	x	x	
x	x	x	x	x	x	x	

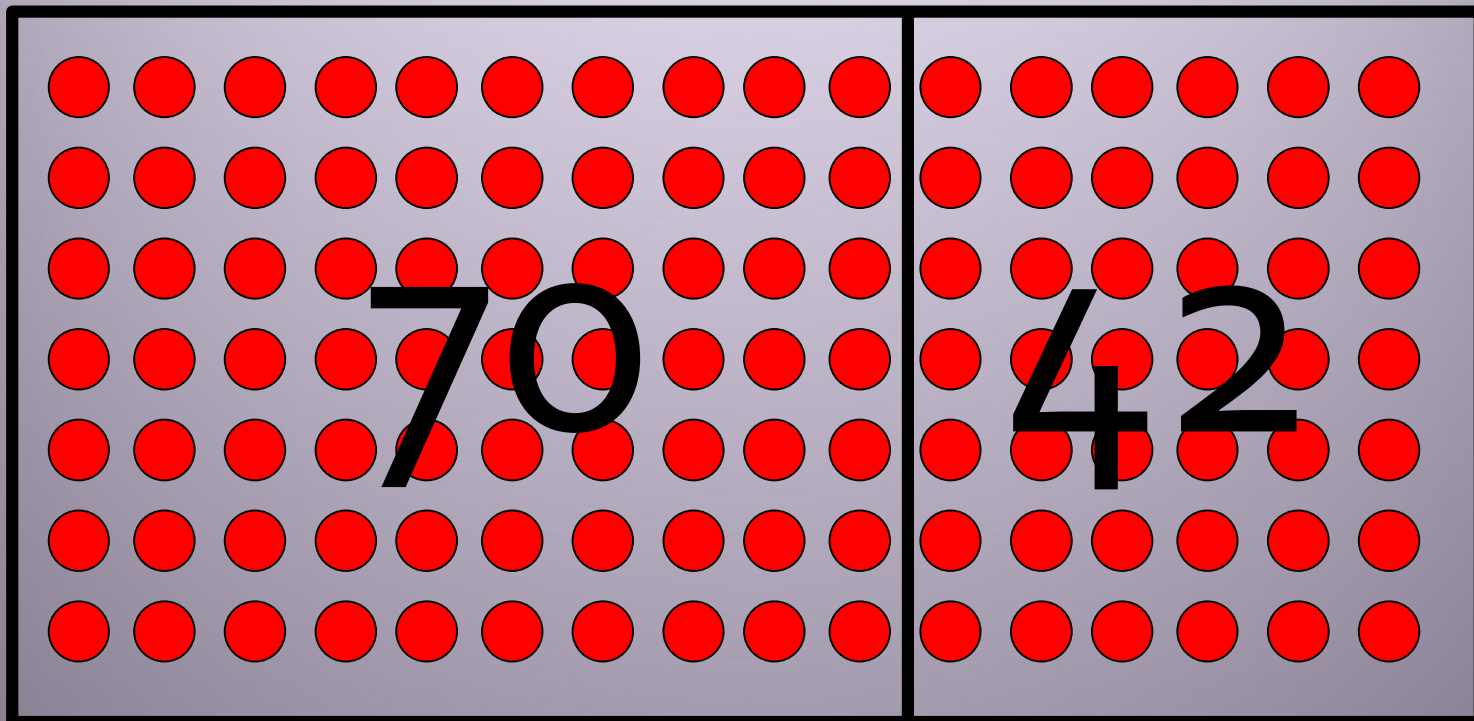
$$4 \times 7 = 7 + 7 + 7 + 7 = 28$$

Arrays to the Grid Method

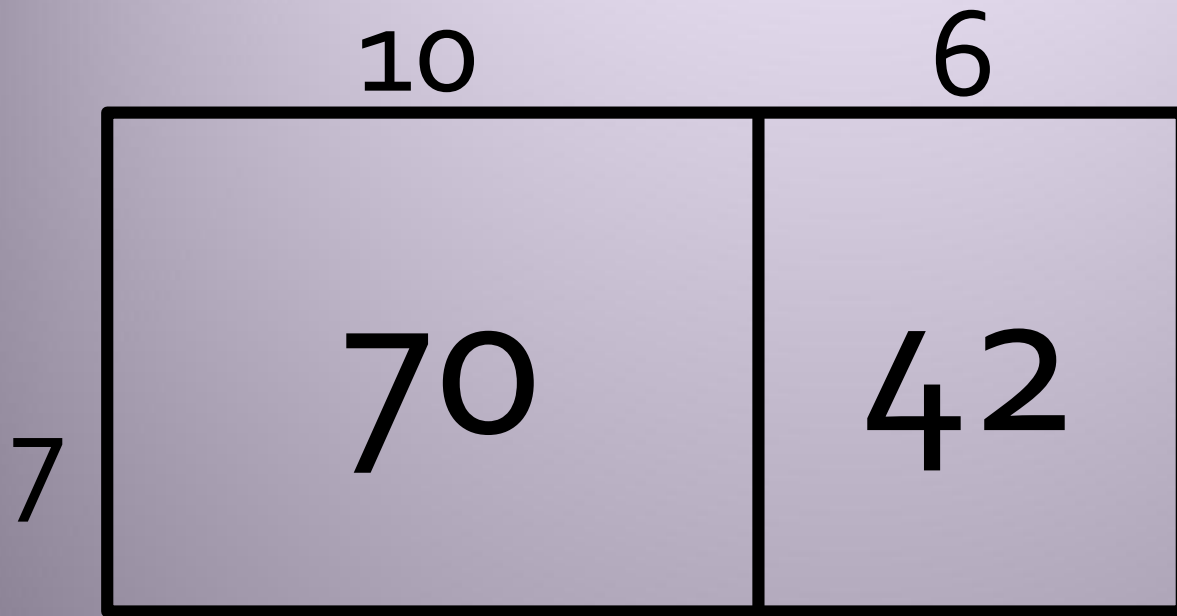
10

6

7



Grid Method



$$\begin{array}{r} 70 \\ + 42 \\ \hline 112 \\ \hline \end{array}$$

Grid Method

Children have to develop their understanding of related facts.

e.g. 23×35

x	20	3
30	600	90
5	100	15

$$\begin{array}{r} 600 \\ 100 \\ 90 \\ + 15 \\ \hline 805 \\ \hline \end{array}$$

Division

Children need to understand the concept of division, that it is:

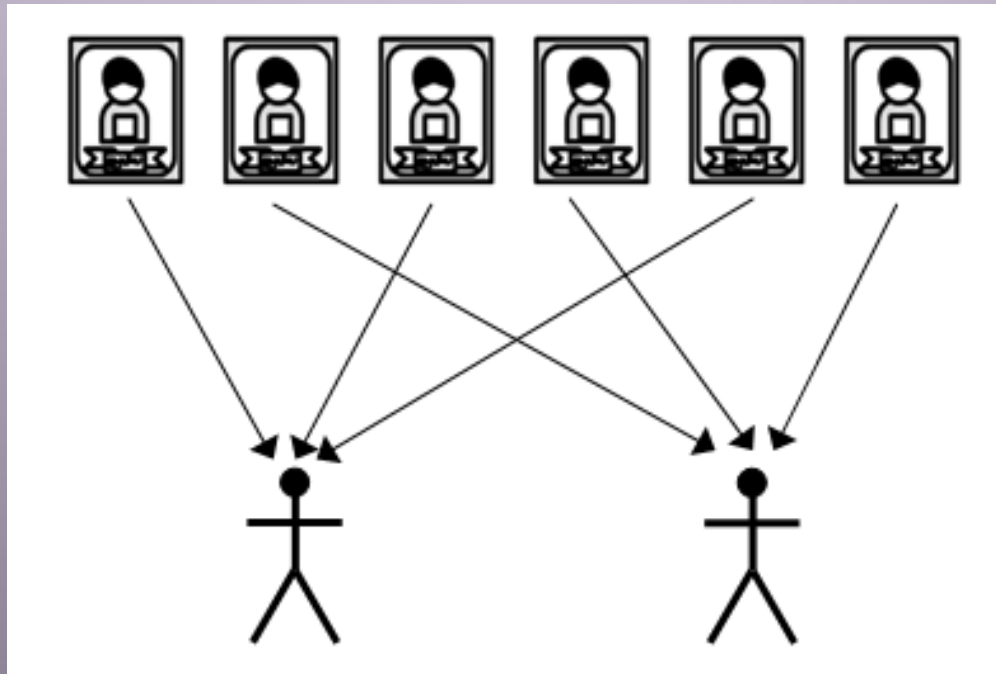
- **Repeated subtraction**

They also need to understand and work with certain principles:

- **Inverse of multiplication**
- **Is not commutative i.e. $15 \div 3 \neq 3 \div 15$**
- **Is not associative i.e. $30 \div (5 \div 2) \neq (30 \div 5) \div 2$**

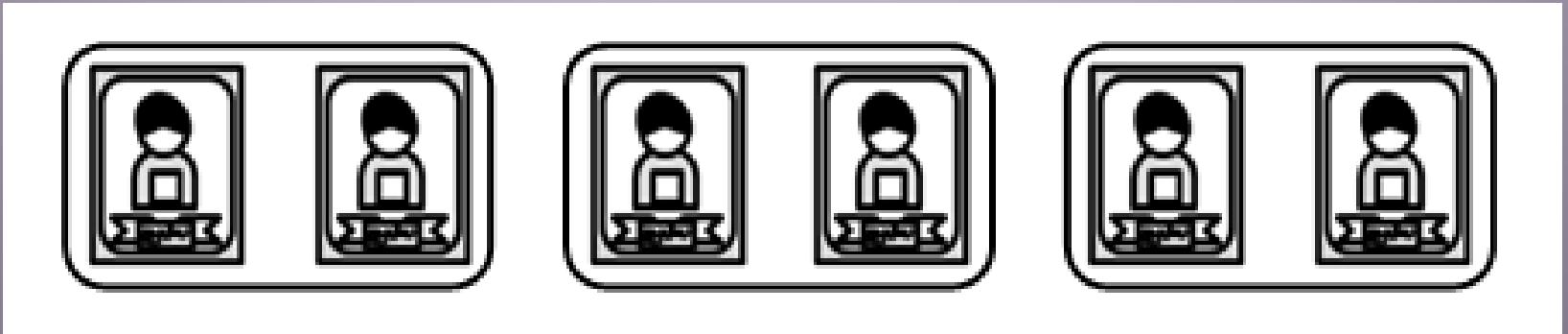
Division as Sharing

Children naturally start their learning of division as division by sharing, e.g. $6 \div 2$.



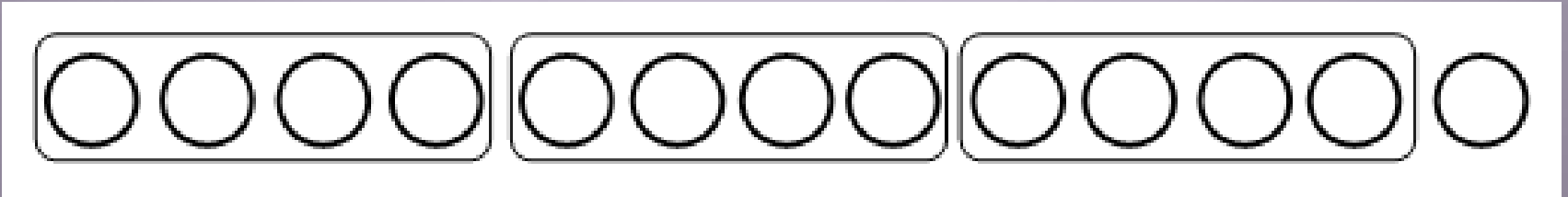
Division as Grouping

To become more efficient, children need to develop the understanding of division as grouping, e.g. $6 \div 2$.



Division as Grouping

To continue their learning, children need to understand that division calculations sometimes have remainders, e.g. $13 \div 4$.



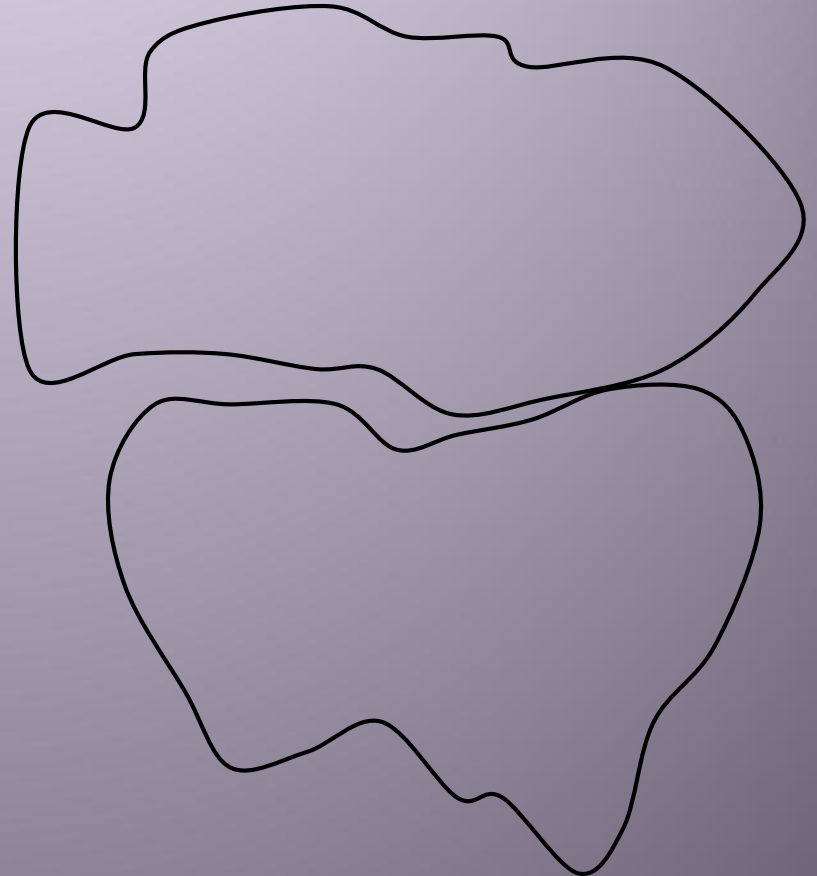
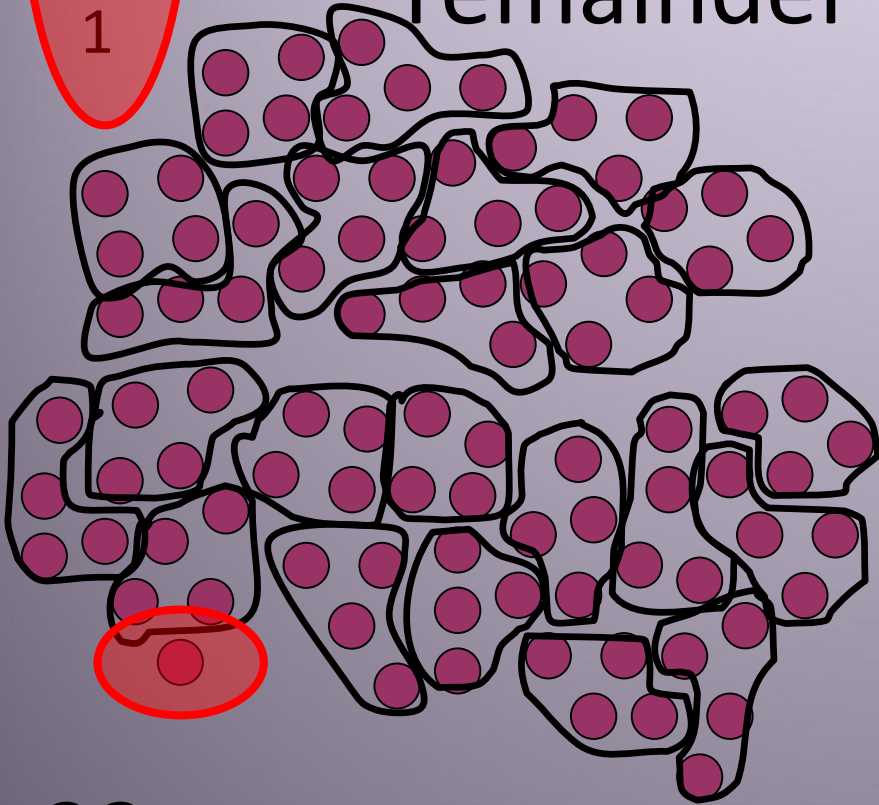
They also need to develop their understanding of whether the remainder needs to be rounded up or down depending on the context.

10
10
1
1
1

Add these together
to find your answer.

$$93 \div 4 = ?$$

remainder 1



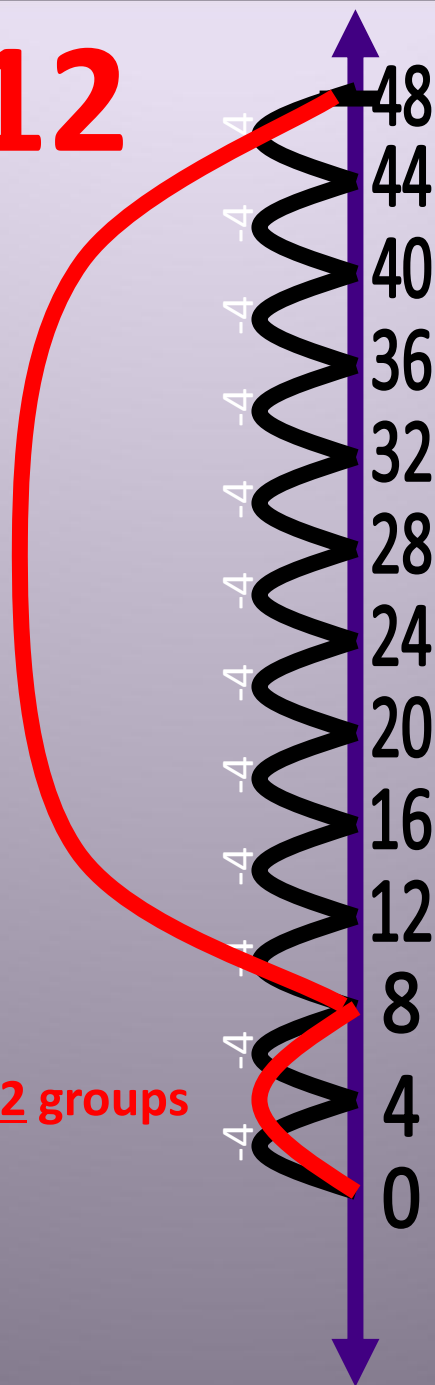
23

Don't forget the
remainder!

$$48 \div 4 = 12$$

10 groups

2 groups



Division by Chunking

Recall of multiplication tables helps make this method more efficient, e.g. $72 \div 3$.

$$\begin{array}{r} 24 \\ 3 \overline{) 72} \\ - 30 \\ \hline 42 \\ - 30 \\ \hline 12 \\ - 6 \\ \hline 6 \\ - 6 \\ \hline 0 \end{array}$$

Answer: 24

1x	3
2x	6
5x	15
10x	30

Children should write key facts in a menu box. This will help them in identifying the largest group they can subtract in one chunk.

Division by Chunking

e.g. $196 \div 6$

$196 \div 6$

$$\begin{array}{r} \underline{32 \text{ r } 4} \\ 6 \overline{) 196} \\ - 180 \\ \hline 16 \\ - 12 \\ \hline 4 \end{array}$$



1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

The key facts in the menu box should be extended to include 4x and 20x.

Answer: 32 remainder 4 or 32 r 4

Key Messages

- For written calculations it is essential that there is a progression which culminates in one method.
- The individual steps within the progression are important in scaffolding children's understanding and should not be rushed through.
- **Practical equipment, models and images are crucial in supporting children's understanding.**